Journal of Applied Mechanics and Technical Physics, Vol. 40, No. 3, 1999

PROPAGATION OF DISTURBANCES IN THREE-DIMENSIONAL SUPERSONIC BOUNDARY LAYERS

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UDC 532.526

The propagation of disturbances in three-dimensional boundary layers under the conditions of a global and a local strong inviscid-viscous interaction is analyzed. A system of subcharacteristics is found based on the condition for the pressure-related subcharacteristic, and an algebraic relation that gives the propagation velocity of disturbances is obtained. The velocity of propagation of disturbances is calculated for two- and three-dimensional flows. The studied problem is of great importance for accurately formulating problems for three-dimensional unsteady boundary-layer equations and for constructing adequate computational models.

Introduction. The development of disturbances is a constituent of the problem of hydrodynamic stability. Analysis of disturbance propagation in a boundary layer corresponds to studying stability against long-wave disturbances. This analysis is required for accurately formulating various problems for three-dimensional unsteady boundary layer equations and for constructing various computational models.

Analyzing three-dimensional boundary layer equations, Wang showed [1, 2] that, in this case, the characteristics are the lines normal to the streamlined surface. This property of the characteristics is related to higher-order derivatives which describe the propagation of disturbances with infinite velocities in the direction normal to the surface.

In [1, 2], based on analysis of the characteristics and the subcharacteristics, Wang gave an accurate formulation of the problem for two- and three-dimensional unsteady boundary layers. A study of the boundary-layer flow with return streams allowed the authors of [3] to separate a discontinuous solution formed during an unsteady process [3].

For the case where the pressure distribution is unknown and should be determined in the course of the solution of the problem, there is an additional mechanism of disturbance propagation which is related to the propagation of pressure waves. The possibility of the upstream propagation of these waves in supersonic and hypersonic-flow regions stems from the existence of a subsonic-flow region near the surface. The induced pressure distribution, which is unknown in advance, is characteristic of the processes of inviscid-viscous interaction. The strong linear interaction processes occurring during the reflection of a shock from a boundary layer was studied by Lighthill [4]. The role of these process was found to be substantial also for separation flows, flows with high local gradients [5, 6], and hypersonic boundary-layer flows [7, 8].

Analysis of disturbance propagation in three-dimensional boundary layers for a regime of strong hypersonic inviscid-viscous interaction allowed the author [9] to determine the subcharacteristic surfaces which separate the region of subcritical (subsonic in the mean) flow from the region of transcritical (supersonic in the mean) flow. According to [10], these flows are called subcritical flows if the disturbances in them can propagate upstream over distances far exceeding the boundary-layer thickness, and transcritical flows if the disturbances in them can propagate only over distances comparable with this thickness.

Two-dimensional unsteady flows were studied by Lipatov in [11–13]. The spatial unsteady propagation of disturbances is the subject of the present study. Two states which correspond to the global and local regimes

Moscow Physicotechnical Institute, Zhukovskii 140160. [1]Central Aerohydrodynamic Institute, Zhukovskii 140160. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 40, No. 3, pp. 116–127, May–June, 1999. Original article submitted March 19, 1997; revision submitted June 24, 1997.

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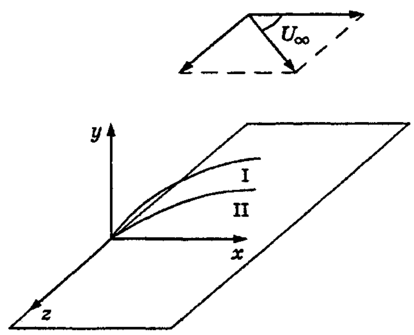

Fig. 1

of strong interaction are studied. In the first case, the disturbances propagate over a distance comparable with the characteristic length of a body. This regime is observed when the parameter of the hypersonic interaction [14] assumes large values [7]. The second regime is characterized by small values of this parameter. In this case, the disturbances propagate over distances which are small compared with the body length but exceed the characteristic boundary-layer thickness.

1. Global Strong Interaction. We consider a hypersonic flow of a viscous gas around a flat zero-incidence surface in a regime of strong hypersonic inviscid–viscous interaction [14] under the conditions

$$\mathrm{M}_\infty \to \infty, \qquad \mathrm{M}_\infty \tau \to \infty, \tag{1.1}$$

where M_∞ is the Mach number of the undisturbed inviscid flow and τ is the dimensionless boundary-layer thickness.

The following notation is used for the Cartesian coordinates reckoned in the direction of free flow, along the normal to the surface, and in the transverse direction, for the time, and the corresponding velocity components, density, pressure, total enthalpy, and dynamic viscosity: lx, ly, lz, lt/u_∞, $u_\infty u$, $u_\infty v$, $u_\infty w$, $\rho_\infty \rho$, $\rho_\infty u_\infty^2 p$, $u_\infty^2 H/2$, and $\mu_0 \mu$. The parameter l is the characteristic length (for example, the length of the generatrix stretching in the streamwise direction), $\tau = (\rho_\infty u_\infty l/\mu_0)^{-1/4}$ (the subscript ∞ denotes the dimensional quantities in free flow), and μ_0 is the dynamic viscosity at the stagnation temperature. The gas is assumed to be thermodynamically perfect and characterized by a constant ratio of specific heats γ. Although, in a hypersonic flow, the effects of a real gas can be significant, they are ignored in our consideration, since the inclusion of them does not change substantially the form of the relationships obtained below. The Reynolds number is large, but it does not exceed the critical value at which the laminar–turbulent transition occurs. The Reynolds number for super- and hypersonic flows is known to be sufficiently large [15].

In accord with the theory of strong interaction [14], the disturbed flow region can be divided into two subregions: a shock-layer subregion I, and a boundary-layer subregion II (Fig. 1). The specific regions near the leading edge and in the temperature transient layer in the vicinity of the outer boundary-layer region are not considered here, since the flow in this region does not affect, in a first-order approximation, the boundary-layer flow.

In subregion I, the flow and coordinate functions [14] are presented in the form

$$(p, \rho, H) = (\tau^2 p_1 + \ldots,\ \rho_1 + \ldots,\ H_1 + \ldots).$$

Substitution of the above expansions into the system of Navier–Stokes equations and the limiting transition (1.1) yield the system of nonlinear equations which describes a disturbed inviscid flow in the shock layer [14]

$$\frac{\partial \rho_1}{\partial t_1} + \frac{\partial \rho_1}{\partial x_1} + \frac{\partial \rho_1 v_1}{\partial y_1} = 0, \quad \frac{\partial v_1}{\partial t_1} + \frac{\partial v_1}{\partial x_1} + v_1 \frac{\partial v_1}{\partial y_1} + \frac{1}{\rho_1}\frac{\partial p_1}{\partial y_1} = 0, \quad \frac{\partial}{\partial x_1}\left(\frac{p_1}{\rho_1^\gamma}\right) + v_1 \frac{\partial}{\partial y_1}\left(\frac{p_1}{\rho_1^\gamma}\right) = 0$$

with the boundary conditions in the shock wave

$$y_1 = g_1(x_1, t_1), \quad \rho_1 = \frac{\gamma+1}{\gamma-1}, \quad p_1 = \frac{(\gamma+1)v_1^2}{2}, \quad v_1 = \frac{2}{\gamma+1}\left(\frac{\partial g_1}{\partial x_1} + \frac{\partial g_1}{\partial t_1}\right)$$

and at the outer border of the boundary layer

$$y_1 = \delta_1(x_1, t_1), \qquad v_1 = \frac{2}{\gamma+1}\left(\frac{\partial \delta_1}{\partial x_1} + \frac{\partial \delta_1}{\partial t_1}\right).$$

For further analysis, it is required to establish the relation between the boundary-layer thickness (or the vertical velocity at the outer border of the boundary layer) and the induced pressure disturbance. We use the following approximate relation:

$$p_1 = \frac{\gamma+1}{2} v_1^2, \tag{1.2}$$

which is an extension of the tangent-wedge formula to the nonstationary case.

For subregion II, the following asymptotic representations are characteristic:

$$(x, y, z, t) = (x_1, \tau y_1, z_1, t_1); \tag{1.3}$$

$$(u, v, w) = (u_2 + \ldots, \tau v_2 + \ldots, w_2 + \ldots), \quad (p, \rho, H) = (\tau^2 p_2 + \ldots, \tau^2 \rho_2 + \ldots, H_2 + \ldots). \tag{1.4}$$

Substitution of (1.3) and (1.4) into the system of Navier–Stokes equations and the limiting transition (1.1) result in the following system for a three-dimensional unsteady boundary layer:

$$\begin{gathered}
X\frac{\partial U}{\partial T} + X\left(U\frac{\partial U}{\partial X} - \frac{\partial F}{\partial X}\frac{\partial U}{\partial Y} - \frac{\partial \Phi}{\partial Z}\frac{\partial U}{\partial Y}\right) - \frac{F}{4}\frac{\partial U}{\partial Y} + \beta\frac{\gamma-1}{4\gamma}Q = \frac{P}{C_0}\frac{\partial^2 U}{\partial Y^2}, \\
X\frac{\partial W}{\partial T} + X\left(U\frac{\partial W}{\partial X} - \frac{\partial F}{\partial X}\frac{\partial W}{\partial Y} - \frac{\partial \Phi}{\partial Z}\frac{\partial W}{\partial Y}\right) - \frac{F}{4}\frac{\partial W}{\partial Y} + \beta\frac{\gamma-1}{4\gamma}Q = \frac{P}{C_0}\frac{\partial^2 W}{\partial Y^2}, \\
X\frac{\partial G}{\partial T} + X\left(U\frac{\partial G}{\partial X} - \frac{\partial F}{\partial X}\frac{\partial G}{\partial Y} - \frac{\partial \Phi}{\partial Z}\frac{\partial G}{\partial Y}\right) - \frac{F}{4}\frac{\partial G}{\partial Y} = XQ\frac{\gamma-1}{\gamma}\frac{\partial P}{\partial T} + \frac{P}{C_0}\frac{\partial^2 G}{\partial Y^2}, \\
\frac{\partial P}{\partial Y} = 0, \qquad \beta = -1 + \frac{2X}{P}\frac{\partial P}{\partial X}, \qquad \Delta = \left[\frac{(\gamma-1)C_0}{2\gamma P^2}\right]^{1/2}\int_0^\infty Q^2\,dY, \\
P = \frac{\gamma+1}{2}\left[\frac{3\Delta}{4} + X\left(\frac{\partial \Delta}{\partial X} + \frac{\partial \Delta}{\partial T}\right)\right]^2, \qquad Q = G - U^2 - W^2.
\end{gathered} \tag{1.5}$$

Here $Y = \left[\frac{\gamma-1}{2\gamma C_0}\right]^{1/2} x_1^{-1/4}\int_0^{y_1} R\,dy_1$, $\delta_1 = x_1^{3/4}\Delta$, $u_2 = U = \frac{\partial F}{\partial Y}$, $w_2 = W = \frac{\partial \Phi}{\partial Y}$, $X = x_1$, $Z = z_1$, $T = t_1$; $p_2 = x_1^{-1/2}P$, $\rho_2 = x_1^{-1/2}R$, $C_0 = P(0, T)$, and $G = H_2$.

We note that, in the boundary-layer approximation, the equation for the transverse component of the momentum degenerates, which shows that the functions P and β are independent of the transverse coordinate Y. Without loss of generality, one can assume that the viscosity depends linearly on the temperature, and the Prandtl number equals unity. The solution of the system of equations should satisfy the following boundary conditions at the surface and the outer border of the boundary layer:

$$U = F = \Phi = 0, \quad G = g_w, \quad Y = 0, \quad U = \cos\alpha, \quad W = \sin\alpha, \quad G = 1, \quad Y = \infty. \tag{1.6}$$

Here α is the angle between the direction of the main stream and the OX axis. To determine the unique solution, one should set an additional condition at a certain line downstream of the leading edge, e.g., the bottom pressure at the leading edge [7]. This condition is related to the upstream propagation of disturbances and their influence on the boundary-layer flow:

$$P[X, Z = \lambda(X), T] = \varphi(X, T). \tag{1.7}$$

This pressure disturbance is given by the solution of the problem which describes the flow in the bottom region. At the same time, in the case of a flow around a plate of zero thickness, the unique solution of the problem considered is given by the smoothness condition in the transition from a subsonic to a supersonic (in the mean) flow in the wake [16, 17].

2. Determination of Subcharacteristic Surfaces. The characteristic (subcharacteristic) surface $\Omega(X, Z, T)$ related to the function $P(X, Z, T)$ is the surface at which the derivative $\partial P/\partial\Omega$ is not determined. The procedure used below is only applicable under conditions where the pressure distribution in the system of boundary-layer equations is unknown in advance and is determined in the course of the solution. To do this, the system of equations should contain the additional interaction condition (1.2) and the additional boundary conditions (1.7). Among problems of this type is the problem of disturbance propagation in channels, where the continuity equation written in integral form is used to determine the pressure distribution.

After introducing the new variables $X, Y, Z, T \to \Omega(X, Z, T), Y, Z, T$, the boundary-value problem (1.5)–(1.7) takes the form

$$\frac{\partial U}{\partial\Omega}\Big(\frac{\partial\Omega}{\partial T}+U\frac{\partial\Omega}{\partial X}+W\frac{\partial\Omega}{\partial Z}\Big)-\frac{\partial U}{\partial Y}\Big(\frac{\partial F}{\partial\Omega}\frac{\partial\Omega}{\partial X}+\frac{\partial\Phi}{\partial\Omega}\frac{\partial\Omega}{\partial Z}\Big)+A_1\frac{\partial P}{\partial\Omega}\frac{\partial\Omega}{\partial X}=B_u,$$

$$\frac{\partial W}{\partial\Omega}\Big(\frac{\partial\Omega}{\partial T}+U\frac{\partial\Omega}{\partial X}+W\frac{\partial\Omega}{\partial Z}\Big)-\frac{\partial W}{\partial Y}\Big(\frac{\partial F}{\partial\Omega}\frac{\partial\Omega}{\partial X}+\frac{\partial\Phi}{\partial\Omega}\frac{\partial\Omega}{\partial Z}\Big)+A_2\frac{\partial P}{\partial\Omega}\frac{\partial\Omega}{\partial Z}=B_w, \qquad (2.1)$$

$$\frac{\partial G}{\partial\Omega}\Big(\frac{\partial\Omega}{\partial T}+U\frac{\partial\Omega}{\partial X}+W\frac{\partial\Omega}{\partial Z}\Big)-\frac{\partial G}{\partial Y}\Big(\frac{\partial F}{\partial\Omega}\frac{\partial\Omega}{\partial X}+\frac{\partial\Phi}{\partial\Omega}\frac{\partial\Omega}{\partial Z}\Big)+A_3\frac{\partial P}{\partial\Omega}\frac{\partial\Omega}{\partial T}=B_g;$$

$$P=\frac{\gamma+1}{2}\Big[X\frac{\partial\Delta}{\partial\Omega}\Big(\frac{\partial\Omega}{\partial X}+\frac{\partial\Omega}{\partial T}\Big)+\frac{3\Delta}{4}+X\frac{\delta\Delta}{\partial T}\Big]^2,$$
$$A_1=A_2=\frac{(\gamma-1)Q}{2\gamma P},\quad A_3=\frac{(\gamma-1)Q}{\gamma P}. \qquad (2.2)$$

Below, the expressions for the right-hand parts of Eqs. (2.1) are not used. Using the determination of the boundary thickness of displacement of the layer, one can write the derivative on the right-hand side of expression (2.2):

$$\frac{\partial\Delta}{\partial\Omega}=\Big[\frac{(\gamma-1)C_0}{2\gamma P^2}\Big]^{1/2}\Big[\int_0^\infty\frac{\partial Q}{\partial\Omega}\,dY-\frac{1}{P}\frac{\partial P}{\partial\Omega}\int_0^\infty Q\,dY\Big].$$

To find the derivative $\partial Q/\partial\Omega$, we transform system (2.1) to one equation for the function

$$D=\frac{\partial F}{\partial\Omega}\frac{\partial\Omega}{\partial X}+\frac{\partial\Phi}{\partial\Omega}\frac{\partial\Omega}{\partial Z}.$$

Adding the first equation multiplied by $\partial\Omega/\partial X$ to the second equation multiplied by $\partial\Omega/\partial Z$, we obtain the equation

$$A_0\frac{\partial D}{\partial Y}+D\frac{\partial A_0}{\partial Y}+B_1Q\frac{\gamma-1}{2\gamma P}\frac{\partial P}{\partial\Omega}=B_0, \qquad (2.3)$$

where $A_0=\dfrac{\partial\Omega}{\partial T}+U\dfrac{\partial\Omega}{\partial X}+W\dfrac{\partial\Omega}{\partial Z}$ and $B_1=\Big(\dfrac{\partial\Omega}{\partial X}\Big)^2+\Big(\dfrac{\partial\Omega}{\partial Z}\Big)^2$.

Equation (2.3) has the solution

$$D=-B_1A_0\frac{\gamma-1}{2\gamma P}\frac{\partial P}{\partial\Omega}\int_0^Y\frac{Q}{A_0^2}\,dY+A_0\int_0^Y\frac{B_0}{A_0^2}\,dY,$$

which yields the following expressions for the derivatives:

$$\frac{\partial U}{\partial\Omega}=\frac{1}{A_0}\Big(D\frac{\partial U}{\partial Y}-A_1\frac{\partial P}{\partial\Omega}\frac{\partial\Omega}{\partial X}+B_u\Big),\qquad \frac{\partial W}{\partial\Omega}=\frac{1}{A_0}\Big(D\frac{\partial W}{\partial Y}-A_2\frac{\partial P}{\partial\Omega}\frac{\partial\Omega}{\partial Z}+B_w\Big),$$

$$\frac{\partial G}{\partial \Omega} = \frac{1}{A_0}\Big(D\frac{\partial G}{\partial Y} + A_3\frac{\partial P}{\partial \Omega}\frac{\partial \Omega}{\partial T} + B_g\Big).$$

Finally, we have the following expression for the derivative of the induced pressure along the direction perpendicular to the subcharacteristic surface:

$$\frac{\partial P}{\partial \Omega} = PB_p\Big/\Big(N\Big(\frac{\partial \Omega}{\partial X} + \frac{\partial \Omega}{\partial Z}\Big)\Big), \quad N = \frac{\gamma - 1}{2}\Big[\Big(\frac{\partial \Omega}{\partial X}\Big)^2 + \Big(\frac{\partial \Omega}{\partial Z}\Big)^2\Big]I_1 - I_0,$$
$$I_0 = \int_0^\infty (G - U^2 - W^2)\,dY, \quad I_1 = \int_0^\infty (G - U^2 - W^2)^2\Big/\Big(\frac{\partial \Omega}{\partial T} + U\frac{\partial \Omega}{\partial X} + W\frac{\partial \Omega}{\partial Z}\Big)^2 dY. \tag{2.4}$$

After introducing the displacement velocity of the characteristic surface

$$a_x = a\cos\omega = -\frac{1}{B_1}\frac{\partial \Omega}{\partial X}\frac{\partial \Omega}{\partial T}, \qquad a_z = a\sin\omega = -\frac{1}{B_1}\frac{\partial \Omega}{\partial Z}\frac{\partial \Omega}{\partial T},$$

where ω is the angle between the OX axis and the direction of disturbances propagation in the plane XZ, the expression for N in (2.4) takes the form

$$N = \frac{\gamma - 1}{2}\int_0^\infty \frac{(G - U^2 - W^2)^2}{(a - U\cos\omega - W\sin\omega)^2}\,dY - \int_0^\infty (G - U^2 - W^2)\,dY.$$

The characteristic surface $\Omega(X, Z, T)$ is specified by the condition

$$N = 0. \tag{2.5}$$

In a particular case, this condition leads to the expressions obtained for two-dimensional steady [18], three-dimensional steady [9], and two-dimensional unsteady [11] flows. Equation (2.5) gives the mean velocity of disturbances provided that the velocity vector and the enthalpy profiles are known. Although formula (2.5) is derived using a number of assumptions concerning the conditions of the interaction, one can show that this formula is valid for other regimes of interaction. Moreover, the same expression gives the propagation velocity of disturbances in turbulent boundary layers.

Therefore, if the surface $\Omega(X, Z, T)$ exists, it divides the flow into regions in which disturbances either propagate upstream (the subcritical-flow region) or do not propagate (the transcritical-flow regions). One can see a direct analogy between the problems considered here and the gas-dynamic problems. The transition from a supersonic to a subsonic flow can be accompanied by the formation of a shock. Discontinuous structures can also form upon the transition from a supercritical to a subcritical flow. This transition was discussed for steady [16, 17, 19] and unsteady [13] two-dimensional flows.

It is noteworthy that the subcharacteristic surface, which is common for all unknown functions $\Omega_1(X, Y, Z, T)$, can be determined based on analysis of system (2.1), (2.2), (2.5) written in the form $\|E\|\dfrac{\partial \boldsymbol{S}}{\partial \Omega} = \boldsymbol{B}$, where the matrix $\|E\|$ and the vector $\boldsymbol{S}$ have the form

$$\|E\| = \begin{pmatrix} A_0 & 0 & 0 & 0 & A_1 \\ 0 & A_0 & 0 & 0 & A_2 \\ C_1 & C_2 & C_3 & 0 & 0 \\ 0 & 0 & 0 & A_0 & A_3 \\ 0 & 0 & 0 & 0 & A_4 \end{pmatrix}, \qquad \boldsymbol{S} = \begin{vmatrix} U \\ W \\ V \\ G \\ P \end{vmatrix},$$

$$C_1 = \frac{\partial \Omega}{\partial X}, \quad C_2 = \frac{\partial \Omega}{\partial Z}, \quad C_3 = \frac{\partial \Omega}{\partial Y}, \quad A_4 = N\Big(\frac{\partial \Omega}{\partial X} + \frac{\partial \Omega}{\partial Z}\Big).$$

The subcritical surface is determined by the equation $\det\|E\| = 0$.

3. Results of Numerical Analysis. To determine the velocity of disturbances, one should know the velocity and the enthalpy profiles across the boundary layer. Provided that the amplitude of unsteady disturbances is low and the undisturbed flow in the boundary layer is stationary, one can use Eq. (2.5) to determine the velocity of disturbances, where the velocity and the enthalpy profiles can be found by solving two- and three-dimensional stationary boundary-layer equations. In the general case, this solution can be obtained numerically.

We consider flows in two- and three-dimensional boundary layers in the cases where these flows are described by self-similar solutions. An example is the flow around a flat plate of infinite span with a sharp leading edge located at zero angle of attack to the approach of the hypersonic flow of a viscous heat-conducting gas provided that the pressure distribution in the outside inviscid hypersonic flow is known and depends only on the streamwise coordinate X. It is assumed that either the plate is semi-infinite in the streamwise direction or the pressure, which corresponds to the self-similar solution, is specified at the rear edge of a plate of finite length. After introducing the Dorodnitsyn–Lees variables

$$\xi(X) = \int_0^X \rho_w \mu_w \, d\tilde{x}, \qquad \eta(X, y_1) = \frac{1}{\sqrt{2\xi}\cos\alpha} \int_0^{y_1} \rho \, d\tilde{y},$$

$$f = \int_0^\eta u \, d\eta, \qquad \varphi = \int_0^\eta w \, d\eta, \qquad g = H, \tag{3.1}$$

we have a self-similar system of three-dimensional boundary-layer equations

$$f_{\eta\eta\eta} + f f_{\eta\eta} + \beta(g - f_\eta^2 \cos^2\alpha - \varphi_\eta^2 \sin^2\alpha) = 0,$$

$$\beta = -\frac{\gamma - 1}{\gamma} \frac{n}{n+1} \frac{1}{\cos^2\alpha}, \qquad \tilde{\varphi}_{\eta\eta} + f \tilde{\varphi}_\eta = 0, \tag{3.2}$$

where $\tilde{\varphi} = \varphi_\eta$ and $g_{\eta\eta} + f g_\eta = 0$, with the boundary conditions

$$\eta = 0: \qquad f = f_w, \quad f_\eta = \tilde{\varphi} = 0, \quad g = g_w,$$

$$\eta = \infty: \qquad f_\eta = \tilde{\varphi} = 1, \quad g = 1.$$

Here γ is the adiabatic exponent and n is the power exponent in the pressure-distribution law $p = c_1 X^n$. In this work, we studied two types of flow, which correspond to the induced pressure distribution ($n = -0.5$) and the unfavorable pressure gradient preset in the outside flow ($n > 0$). System (3.2) was written in the linearized form

$$f_k''' + f_{k-1} f_k'' + \beta(g_{k-1} - f_{k-1} f_k \cos^2\alpha - \varphi_{k-1}^2 \sin^2\alpha) = 0,$$

$$\tilde{\varphi}_{k-1}'' + f_{k-1} \tilde{\varphi}_{k-1}' = 0, \qquad g_{k-1}'' + f_{k-1} g_{k-1}' = 0$$

(k is the order of approximation) and was approximated with a second-order difference scheme.

In the case $n = -0.5$, with given boundary conditions, the first equation set was solved by the four-diagonal sweep method. The last two equations have solutions which can be written in analytical form

$$\tilde{\varphi}_{k-1}(\eta) = I(\eta)/I(\infty),$$

where $I(\eta) = \int_0^\eta \exp\Big(-\int_0^{\tilde{\eta}} f_{k-1} \, d\tilde{\eta}\Big) d\tilde{\eta}$ and $g_{k-1}(\eta) = g_w + (1 - g_w)\tilde{\varphi}_{k-1}(\eta)$.

In the case $n > 0$, the region of definition of the desired functions was divided into two segments $[0, \eta^*]$ and $[\eta^*, \eta_\infty]$ (Fig. 2), and the general problem split into two problems: a lower boundary-value problem I with the upper boundary condition $f' = 0$ for the function f at the point η^* and the upper boundary problem II with the lower boundary conditions $f' = 0$, $f = f_{\rm I}(\eta^*)$ for the function f, where $f_{\rm I}(\eta^*)$ is the value of the solution on the segment I at the point η^*.

Next, the difference between the second-order one-sided derivatives at the point η^* was found: $\Delta = f_{\rm I}''(\eta^*) - f_{\rm II}''(\eta^*)$. With the use of an iteration process (e.g., the dichotomy method), the point η^* was chosen so

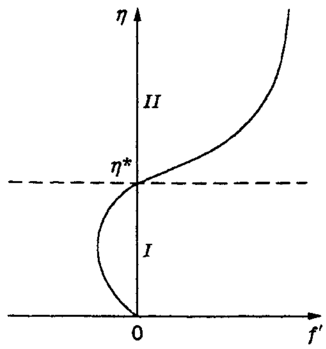

Fig. 2

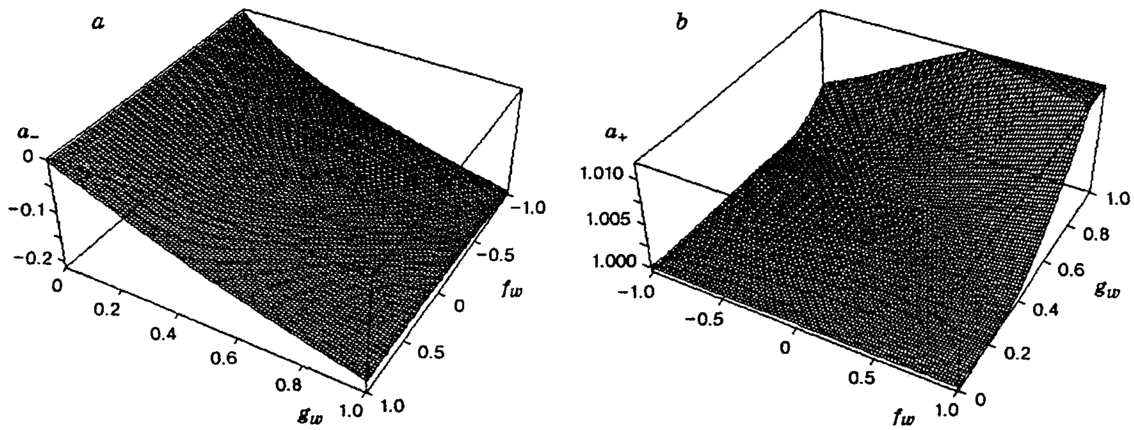

Fig. 3

as to minimize the value of $|\Delta|$ with preset accuracy. The velocity profile determined by the above-described method corresponds to one branch of solution, i.e., a return flow with $f_w'' < 0$ [20].

After determining the velocity and enthalpy profiles, relation (2.5) written in terms of the variables (3.1) was used to find the velocity of disturbances propagation

$$\frac{\gamma-1}{2}\int\limits_0^\infty \frac{(G-U^2-W^2)^2}{(a-U\cos\omega-W\sin\omega)^2}\,d\eta-\int\limits_0^\infty (G-U^2-W^2)\,d\eta=0,$$

where $G = g$, $U = f_\eta \cos\alpha$, and $W = \tilde{\varphi}\sin\alpha$.

Below, the results of numerical and theoretical analysis of disturbance propagation with varied characteristic parameters of the problem for two- and three-dimensional flows are given.

Two-Dimensional Flows. Figure 3a and b shows the graphs of the upstream (a_-) and downstream (a_+) velocities of disturbances in the range of variation of the parameters $f_w \in [-1,1]$ and $g_w \in [0,1]$. The nonzero stream function at the surface ($f_w \neq 0$) corresponds to a power law of distribution of the suction or blowing rate. An increase in the suction rate decreases the velocity a_- owing to the relative decrease in the subsonic-flow region in the boundary layer. The increase in the blowing rate leads to the opposite tendency. The temperature factor g_w equals the ratio between the surface and stagnation temperatures. Heating gives rise to the increase in the boundary-layer thickness and the relative thickness of the subsonic-flow region. As a result, the velocity of the upstream disturbances increases. One can see that, as g_w tends to zero, the velocity of the upstream disturbances also tends to zero. Thus, the disturbances do not propagate upstream at zero surface temperature. A similar result was obtained in [21–23], where the eigenvalues of the two-dimensional stationary boundary-value problem (1.5) were studied.

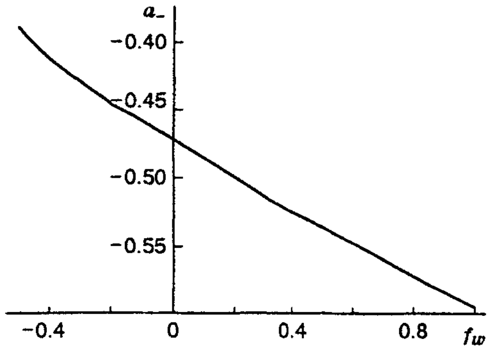

Fig. 4

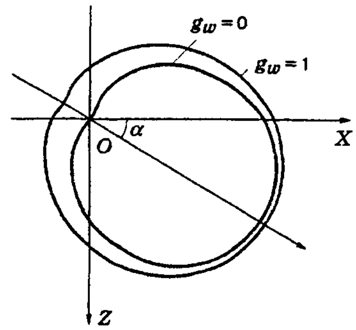

Fig. 5

The self-similar solution of this system for a laminar boundary layer with pressure rise in the outside flow corresponds to the negative values of the parameter β in system (3.1). It is known [20] that the solution of the problem of interest is not unique under these conditions, and one of the branches describes flow with return streams. The occurrence of return flows in the boundary layer gives rise to the convective mechanism of upstream propagation of disturbances. It is important that, in this case, the characteristics of the propagation of pressure disturbances also change. This fact is supported by calculation results. Figure 4 shows the dependence of the velocity of upstream disturbances in a hypersonic boundary layer a_- on the parameter f_w for $g_w = 1$ and a power-law pressure distribution in the outside stream $p = c_1 X^{0.1}$ for a branch of solution of (3.1) that corresponds to the negative surface friction. It should be noted that the formation of return-flow regions appreciably increases the velocity of upstream disturbances.

Three-Dimensional Boundary Layer. The self-similar solution of Eqs. (3.1) describes a boundary-layer flow over a yawed wing of infinite span. This solution was used to find the vector $\boldsymbol{a} = (a_x, a_z)$ as a function of the temperature factor g_w and the incidence angle α (the angle between the OX axis and the approach-stream direction). Figure 5 shows the directional pattern of the propagation velocity of disturbances in a laminar boundary layer over a yawed wing in a regime of strong hypersonic inviscid–viscous interaction for $g_w = 0$ and $\alpha = 30°$ (quantitatively, this pattern is similar to that for a zero-incidence wing [12]).

4. Local Strong Interaction. In this paper, the term "strong interaction" corresponds to a regime in which the boundary-layer flow introduces only small disturbances into an outside inviscid flow. At the same time, the induced disturbances are assumed to affect the boundary-layer flow already in a first-order approximation. This regime of strong interaction can be either global (manifesting itself over the entire body length) or local (manifesting itself over small distances compared with the characteristic length). The regime of strong local interaction between a boundary-layer flow with an outside hypersonic flow is characterized by the following limiting relations: $M_\infty \to \infty$, $M_\infty \tau_1 \to 0$, and $\tau_1 = O((\rho_0 u_\infty l/\mu_0)^{-1/2})$. In this case, the disturbance-induced effect is observed in a short region whose dimensions exceed the boundary-layer thickness [17]. We consider a flow over a plate (or a wedge surface) under the assumption that, at a finite distance from the leading edge, the boundary-layer flow is affected by a source of disturbances (e.g., varied bottom pressure or a jump of varying intensity). The following limiting relations are assumed to be fulfilled [19, 24]:

$$g_w \to 0, \qquad \varepsilon_0 g_w^{2+\omega} M_\infty^{-1} = O(1), \qquad \Delta p M_\infty g_w^{\omega/2} = O(1), \qquad \varepsilon_0 = (\rho_0 u_\infty l/\mu_0)^{-1/2}, \tag{4.1}$$

where Δp is the amplitude of the disturbances and $\mu = C_\mu T^\omega$. As shown in [19, 24], in this interaction regime, a three-scale scheme similar to the scheme arising during the supersonic interaction [5, 6] can be realized. In this case, new effects related to the formation of the total displacement thickness can appear. For example, during strong wall cooling, the displacement thickness of the boundary layer varies not only in the near-wall region, but also in the main region of the boundary-layer flow, provided that conditions (4.1) hold. Under the assumption that the flows in the outside inviscid stream and in the main part of the boundary layer are two-dimensional, the boundary-value problem which describes the flow in the near-wall region takes the form

$$\frac{\partial U_0}{\partial T_0} + U_0 \frac{\partial U_0}{\partial X_0} + V_0 \frac{\partial U_0}{\partial Y_0} + W_0 \frac{\partial U_0}{\partial Z_0} + \frac{\partial P_0}{\partial X_0} = \frac{\partial^2 U_0}{\partial Y_0^2},$$

$$\frac{\partial W_0}{\partial T_0} + U_0 \frac{\partial W_0}{\partial X_0} + V_0 \frac{\partial W_0}{\partial Y_0} + W_0 \frac{\partial W_0}{\partial Z_0} + \frac{\partial P_0}{\partial Z_0} = \frac{\partial^2 W_0}{\partial Y_0^2}, \qquad \frac{\partial U_0}{\partial X_0} + \frac{\partial V_0}{\partial Y_0} + \frac{\partial W_0}{\partial Z_0} = 0,$$

$$Y_0 = 0, \quad U_0 = W_0 = 0, \quad Y_0 \to \infty, \quad U_0 = Y_0 + A + o(1), \quad W_0 = 0,$$

where

$$X_0 = (x-1)(a_0^5 g_w^{(2+\omega)/4} \varepsilon_0^{-3} \mathrm{M}_\infty^3)^{1/4}; \qquad Y_0 = y(a_0^3 g_w^{-(2+\omega)/4} \varepsilon_0^{-5} \mathrm{M}_\infty)^{1/4};$$

$$Z_0 = z(a_0^5 g_w^{(2+\omega)/4} \varepsilon_0^{-3} \mathrm{M}_\infty^3)^{1/4}; \quad T_0 = t(a_0^2 g_w^{(2+\omega)/2} \varepsilon_0^{-2} \mathrm{M}_\infty^2)^{1/4};$$

$$U_0 = u(a_0 g_w^{-(2+\omega)/4} \varepsilon_0^{-1} \mathrm{M}_\infty)^{1/4}; \qquad P_0 = \Big(p - \frac{1}{\gamma \mathrm{M}_\infty^2}\Big)(a_0 g_w^{\omega} \varepsilon_0 \mathrm{M}_\infty^{-1})^{-1/2}.$$

After introducing the new variables $X_0, Y_0, Z_0, T_0 \to \Omega_0(X_0, Z_0, T_0), Y_0, Z_0, T_0$ and some transformations, we obtain

$$\frac{\partial P_0}{\partial \Omega_0} = \frac{P_0}{N_0}, \qquad N_0 = \frac{\partial \Omega_0}{\partial X_0}\Big\{L + \Big[\Big(\frac{\partial \Omega_0}{\partial X_0}\Big)^2 + \Big(\frac{\partial \Omega_0}{\partial Z_0}\Big)^2\Big] I_2\Big\},$$

$$I_2 = \int_0^\infty \Big(\frac{\partial \Omega_0}{\partial T_0} + U_0 \frac{\partial \Omega}{\partial X_0} + W_0 \frac{\partial \Omega_0}{\partial Z_0}\Big)^{-2} dY_0,$$

where $L = (d\tau_1/dp)\, a_0^{5/4} g_w^{(2+\omega)/2} \varepsilon_0 \mathrm{M}_\infty^{-1/4}$.

After introducing the velocity components $a_z = a \sin\omega$ and $a_x = a\cos\omega$, the condition $N_0 = 0$, which gives the characteristic surface, takes the form

$$L + \int_0^\infty \frac{dY_0}{(a - U_0 \cos\omega - W_0 \sin\omega)^2} = 0. \tag{4.2}$$

Thus, provided that the solution of a stationary problem which describes a three-dimensional disturbed flow in the near-wall region is known, one can use formula (4.2) to determine the velocity of disturbance propagation under the conditions of strong local hypersonic interaction.

Conclusions. The effects of disturbance propagation play an important role in the problems of flow sensitivity and stability (generally, they are ignored). Pressure disturbances can change the characteristics of the initial boundary layer. Moreover, conditions under which the upstream and downstream waves can interact with each other can arise. It should be noted that these effects are insignificant in subsonic flows. This is, probably, connected with the fact that the hypersonic boundary layer acts as a waveguide in which the amplitude of the disturbances propagating upstream decays to a considerably lesser extent than in supersonic and subsonic boundary layers. The near-surface subsonic sublayer, in which the disturbances just propagate, plays an important part in the development of disturbances. The results show that, in numerical modeling of viscous-gas hypersonic flows, it is important to accurately reproduce flow not only in the boundary later, but also in the subsonic sublayer. Neglect of the effects associated with disturbances propagation can result in qualitatively incorrect results, computational instability, etc.

In addition, when the disturbances attain a certain high amplitude, a "jump" which separates the regions of supersonic and subsonic flows can occur.

This work was supported by the Russian Foundation for Fundamental Research (Grant 98-01-00660.)

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